EXPERIMENT 4: RC, RL and RD CIRCUITs

Equipment List

- Resistor, one each of
  - 330Ω
  - 1kΩ
  - 1.5kΩ
  - 10kΩ
  - 100kΩ
  - 1000kΩ
- 0.1µF Ceramic Capacitor
- 100µH Inductor
- LED and 1N4001 Diode.

Introduction

We have studied D.C. circuits with resistors and batteries and discovered that Ohm's Law governs the relationship between current through and voltage across a resistance. It is a linear response; i.e.,

$$V = IR \tag{1}$$

Such circuit elements are called passive elements. With A.C. circuits we find other passive elements called capacitors and inductors. These, too, obey Ohm's Law and circuit loop problems can be solved using Kirchoff' 3 Laws in much the same way as with DC. circuits. However, there is one major difference - in an AC. circuit, voltage across and current through a circuit element or branch are not necessarily in phase with one another. We saw an example of this at the end of Experiment 3. In an AC. series circuit the source voltage and current are time dependent such that

$$i(t) = i_s \sin(\omega t)$$
$$v(t) = v_s \sin(\omega t + \phi) \tag{2}$$

where, in this case, voltage leads current by the phase angle ϕ, and V_s and I_s are the peak source voltage and peak current, respectively. As you know or will learn from the text, the phase relationships between circuit element voltages and series current are these:

- resistor - voltage and current are in phase implying that $\phi = 0$
- capacitor - voltage lags current by $90°$ implying that $\phi = -\frac{\pi}{2}$
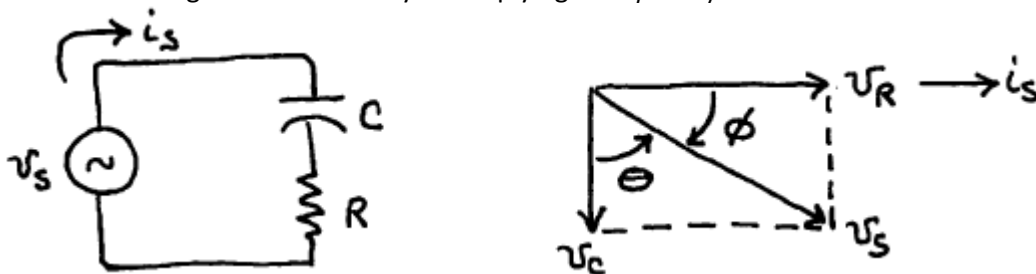- inductor - voltage leads current by $90°$ implying that $\phi = \pi/2$

Figure 1: RC circuit with AC source.

In this experiment we will study a circuit with a resistor and a capacitor in m. Since there is only a single current path through the elements of the circuit, Kirchhoff' s Laws imply that the current must have the same phase in all elements. Thus, the voltages across the resistor and capacitor will be out of phase. Further, we will find that the sum of voltages across the elements will exceed that of the signal generator. To correctly apply Ohm's Law the voltages across the resistor and capacitor must be added vectorally to equal the generator voltage, as shown in Figure 1 above. In this example the voltages and current are shown at t = 0. From the right hand figure we see that v_R leads v_s by $\phi = atan\left(\frac{1}{\omega RC}\right)$ and v_c lags by $\theta = \frac{\pi}{2} - \phi = acot\left(\frac{1}{\omega RC}\right)$. Note that the voltage and current in the resistor are in phase. Then, since the voltage in the capacitor lags that of the current by 90°, the voltage across the capacitor must also lag the voltage across the resistor by 90°.

The equivalent resistance for the capacitor is the capacitive reactance, $X_c = \frac{1}{\omega C}$, where $\omega = 2\pi f$ is called the angular frequency. The **magnitudes** of the voltages across the resistor, capacitor, and source are:

$$v_R = i_s R \quad v_C = i_s X_c = \frac{i_s}{\omega C} \quad v_s = i_s Z \tag{3}$$

Z is called the impedance of the circuit and can be determined from vector addition in Figure 1 to be:

$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} \tag{4}$$

and the phase angle between the voltage and current out of the generator is

$$\phi = \text{atan}\left(\frac{\frac{1}{\omega C}}{R}\right) = \text{atan}\left(\frac{1}{\omega RC}\right) \tag{5}$$

The series RC circuit is seen to be a frequency dependent voltage divider where the output fractions across the resistor and capacitor are

$$\frac{v_R}{v_s} = \frac{R}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} = \frac{1}{\sqrt{1 + \left(\frac{1}{\omega RC}\right)^2}} \tag{6}$$

$$\frac{v_C}{v_s} = \frac{\frac{1}{\omega C}}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} = \frac{1}{\sqrt{1 + (\omega RC)^2}} \tag{7}$$

For low frequencies, most of the generator signal appears across the capacitor,

$$\lim_{\omega RC \to 0} \frac{v_R}{v_s} = 0$$
$$\lim_{\omega RC \to 0} \frac{v_C}{v_s} = 1 \tag{8}$$

while at high frequencies, most appears across the resistor

$$\lim_{\omega RC \to \infty} \frac{v_R}{v_s} = 1$$
$$\lim_{\omega RC \to \infty} \frac{v_c}{v_s} = 0 \quad (9)$$

Thus, for a signal with many frequency components, this voltage divider circuit acts like a filter with the high frequency components passing straight through the capacitor (little voltage drop there) to the resistor and the low frequency components passing straight through the resistor to the capacitor.

- The capacitor output is called a **low pass filter**; i.e., it passes the tow frequencies.
- The resistor output is called a **high pass filter**, i.e. it passes the high frequencies.

An important example of what happens to multifrequency signals can be seen with a square wave input. A square wave of frequency f_o can be synthesized by the summing of sinusoidal signals with frequencies f_o, $2f_o$, $3f_o$. Since a filter will attenuate the different frequencies by different factors, we should expect output waveforms to be distorted. The distortions imposed by high and low pass filters have particular significance.

For the RC circuit we could write v_s lagging i_s by the phase angle ϕ, as in Equations 2, but it is usually the case that the circuit contains a voltage generator, thus, we write the current leading the voltage such that

$$v_s(t) = V_s \sin(\omega t)$$
$$i_s(t) = I_s \sin(\omega t + \phi) \quad (10)$$

Across the resistor, v_R is in phase with the current and leads v_s by ϕ, thus, with respect to v_s,

$$v_R(t) = i_s(t)R = I_s R \sin(\omega t + \phi) = \frac{V_s R}{Z} \sin(\omega t + \phi) \quad (11)$$

Across the capacitor, v_c lags the source voltage v_s, thus

$$v_c(t) = i_s(t) \frac{1}{j\omega C} = \frac{I_s}{\omega C} \sin(\omega t - \theta) = \frac{V_s}{Z\omega C} \sin(\omega t + \phi - \pi/2) \quad (12)$$

Let us now look in the frequency regimes where the signal is reduced across an element.

Differentiation

Look across the resistor, the high pass filter at low frequencies, i.e., when $\omega RC \ll 1$. The phase shift and equivalent circuit impedance, from Eqs. 5 and 4, become $\phi = \frac{\pi}{2}$ and $Z = \frac{1}{\omega C}$. Substituting these values into equation 11 we find

$$v_R(t) = V_s \omega RC \sin\left(\omega t + \frac{\pi}{2}\right) = V_s \omega RC \cos(\omega t) = RC \frac{dv_s(t)}{dt} \quad (13)$$

Thus the high pass filter acts like a differentiator when $\omega RC \ll 1$. Note that the signal will be strongly attenuated (reduced in magnitude).

Attenuation

When a signal is attenuated (negative gain) by a filter it is useful to talk about the degree of attenuation compared to the signal that entered the circuit. One common way to talk about the attenuation is using deciBel:

$$dB = 20 \log_{10} V/V_{ref} \tag{14}$$

The cutoff frequency of a filter is the frequency with a gain of=-3dB.
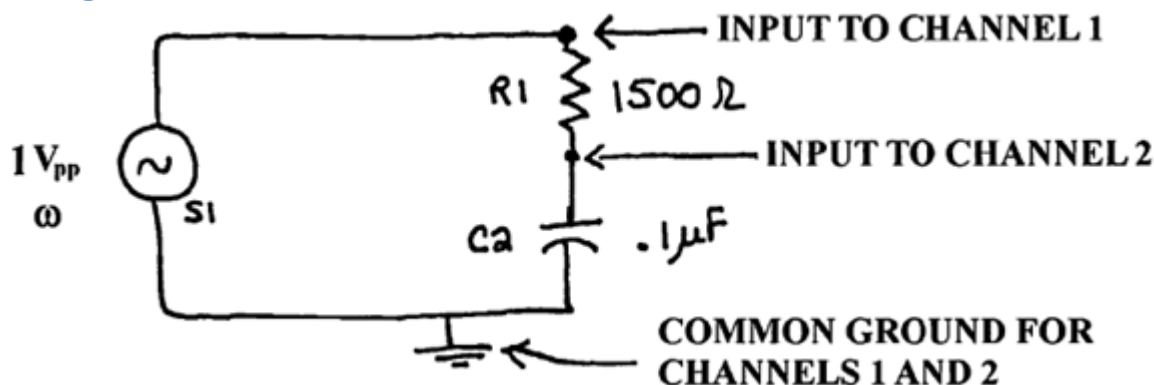
Part I: High and Low Pass RC Filters

Figure 2: Low-pass RC filter.

Construct the circuit in Fig. 2. In this experiment we wish to measure the output fractions given in Eqs. 6 and 7. We will do this by keeping the output voltage-fixed at 1 V_{pp}. Then the fraction at a given frequency is determined by measuring the peak-to-peak voltage across the capacitor and dividing by 1. As you go from low frequency to high frequency the current changes check the terminal voltage of the signal generator to guard against changes. Each time you change frequency you may need to adjust the signal generator to maintain 1 V_{pp}

1. Connect the oscilloscope inputs as shown in Figure 2.

2. Measure V_c for the frequencies given on the data sheet.

3. Reverse the order of the resistor and capacitor so that Channel 2 is across the resistor to ground.

4.   Measure V_R for the frequencies given on the data sheet.
5. Calculate the ratios V_R/V_S and V_c/V_S theoretically from Eqs. 6 and 7.
6. Graph your results using graphing software.
    Question 1: Do the theoretical and experimental values agree?
    Question 2: Find the range of frequencies for which V_c/V_S is at least 0.707 or -3dB.

Part II: High and Low Pass RL Filters

Construct (on your breadboard) the circuit in Fig. 3.

1. Find the range of frequencies for which V_R/V_S is at least 0.707 or -3dB. Hint: one end of the range will either be 0 or infinity.
2. Compare this to the frequency where the impedances of the resistor and inductor are equal in magnitude.
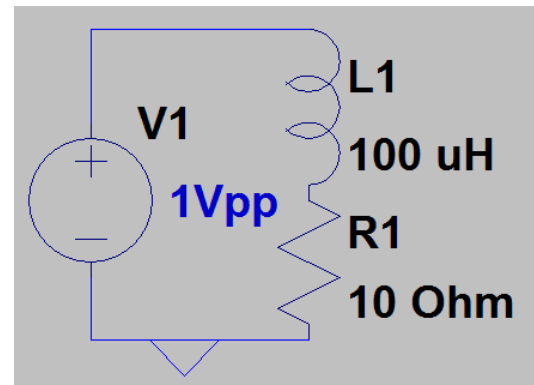
Figure 3: Low pass RL filter circuit with AC source.

Part III RD circuit.

Construct the circuit in Fig. 4.

1N4004 diode for D1

1. On a single graph sketch the voltage across V1 and the voltage across R1 from the oscilloscope for this circuit at 1kHz $5V_{pp}$ AC source.

Figure 4: RD circuit with AC source.

2. Infer the voltage across the diode D1 and sketch and label it on the same graph.
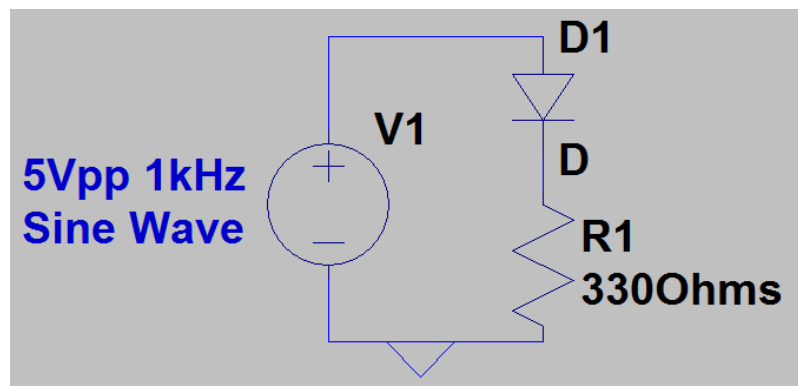3. Reverse the direction of the diode make sketches of the voltage across V1, R1, and D1.

LED for D1

4. Replace the diode with a LED and sketch the current through the diode vs time.
5. Reduce the frequency to 2Hz and use your observations label the points on the graph from step 4 where you infer the diode was illuminated.
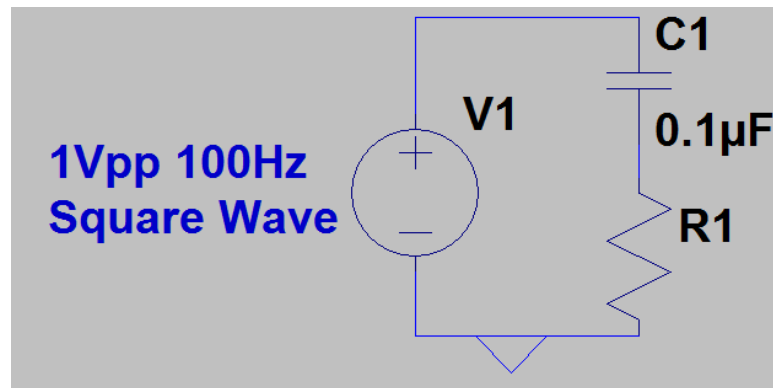
Part IV: Differentiation

Figure 5 High Pass Filter

Construct the circuit in Fig. 5. Use the **square wave** output.

1. Observe the waveforms for the following values 1kΩ, 10kΩ, 100kΩ, 1MΩ. Sketch and label one complete cycle of the observed waveform for each resistance value on the square wave drawn on the datasheet. It will help to display both channels simultaneously.

Name:

Part 1: Include an excel graph of your data (experimental and theoretical)

	Experimental		Theoretical	
Frequency (Hz)	V_c/V_s	V_R/V_s	V_c/V_s	V_R/V_s
50				
100				
500				
1k				
2k				
5k				
10k				
20k				
50k				
100k				

$$\omega = \frac{1}{RC}$$

ω interpolated from measurments:

Question 1:

Question 2:

PART II:

Cutoff frequency and Range:

PART III:

Sketches of $V_1(t)$, $V_R(t)$, and $V_D(t)$ on same graph for Q1&2.

Sketches of $V_1(t)$, $V_R(t)$, and $V_D(t)$ on same graph for Q3.

Labeled sketch of $I_D(t)$ for the LED circuit.

PART IV: DIFFERENTIATION

DIFFERENTIATION